

LLR contribution to long-term EOP variations and celestial reference frames

Jürgen Müller

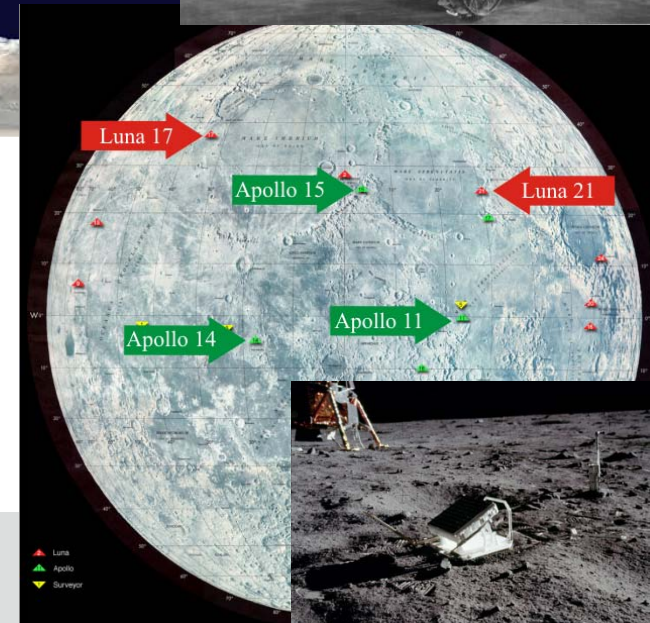
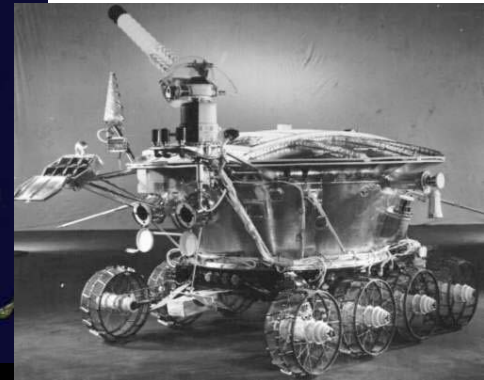
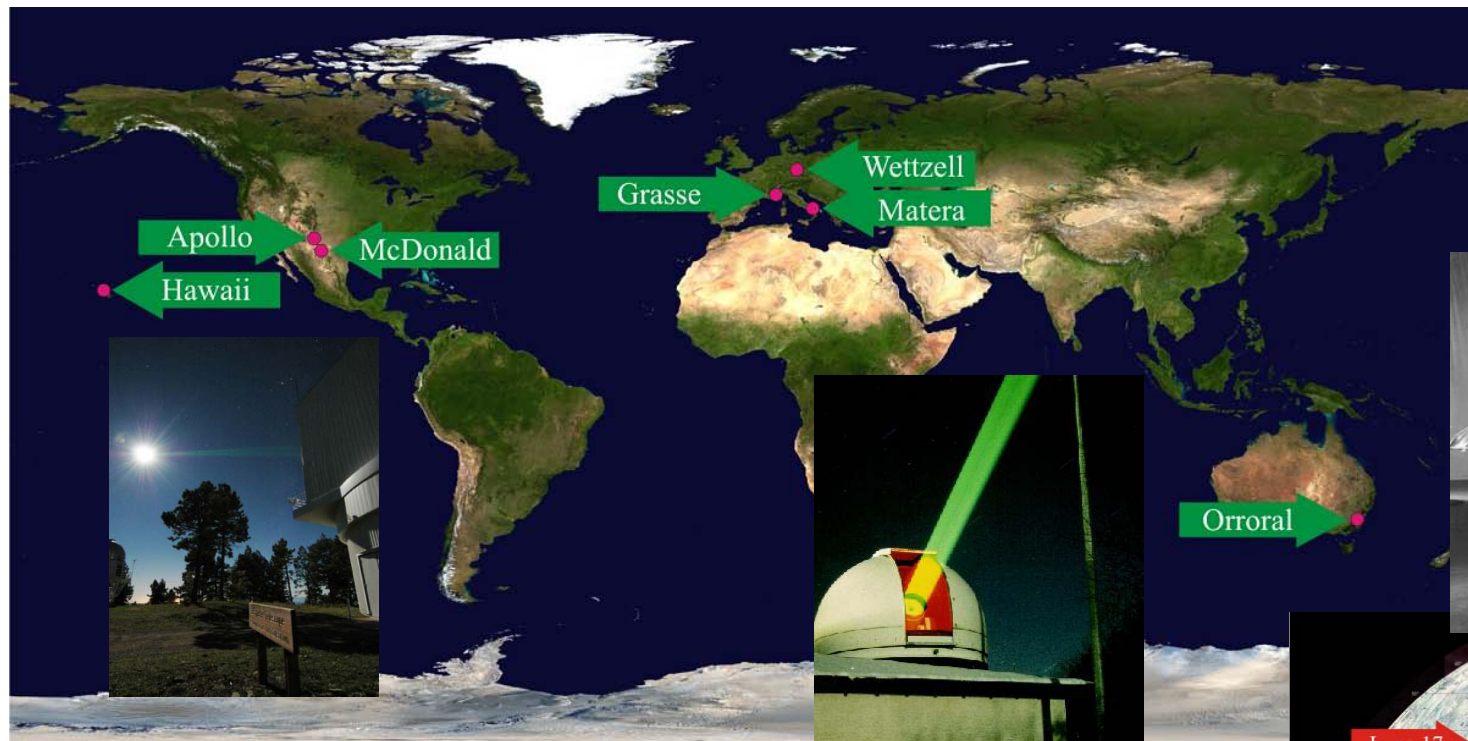
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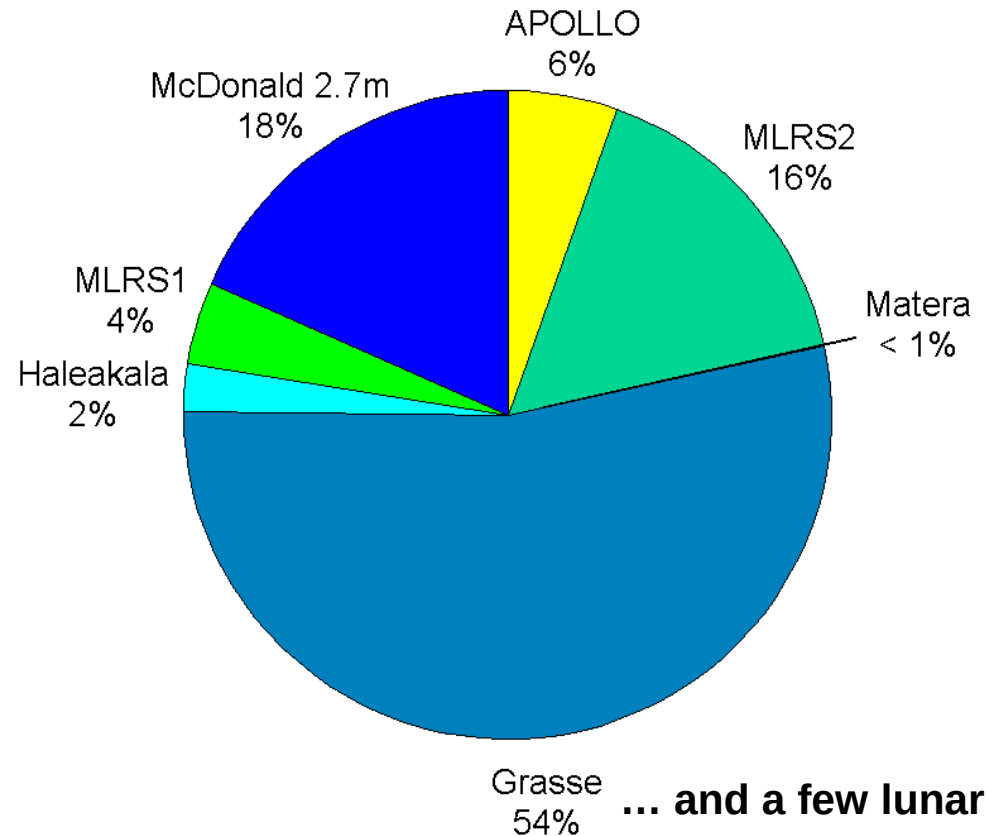
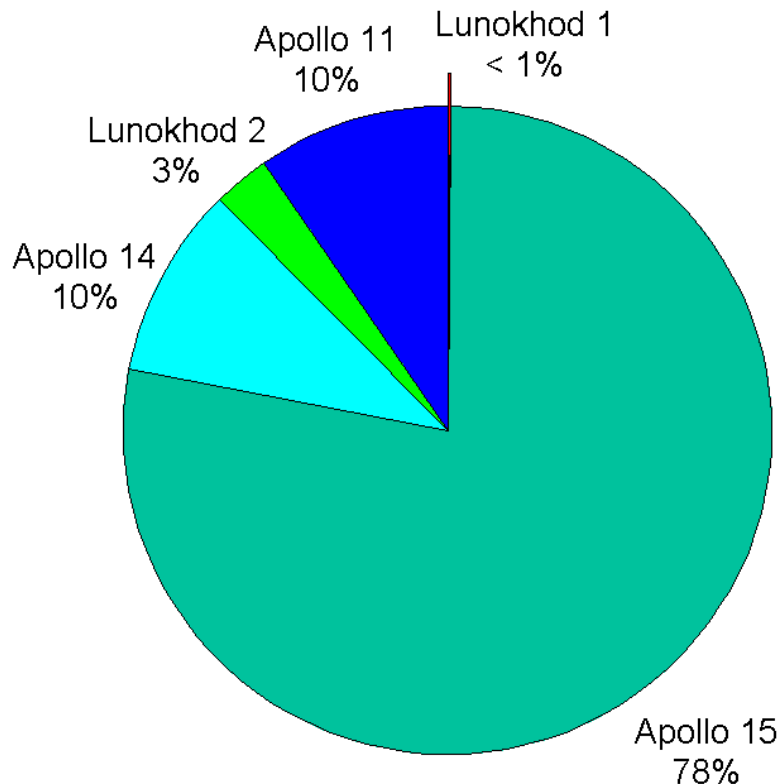
Lunar Laser Ranging



- 42 years of observations
- Post-newtonian model at cm level
- high long-term stability (orbit, reference frames, **Earth orientation**)
- relativity tests

Statistics - reflectors and observatories

Time span 1970-2011

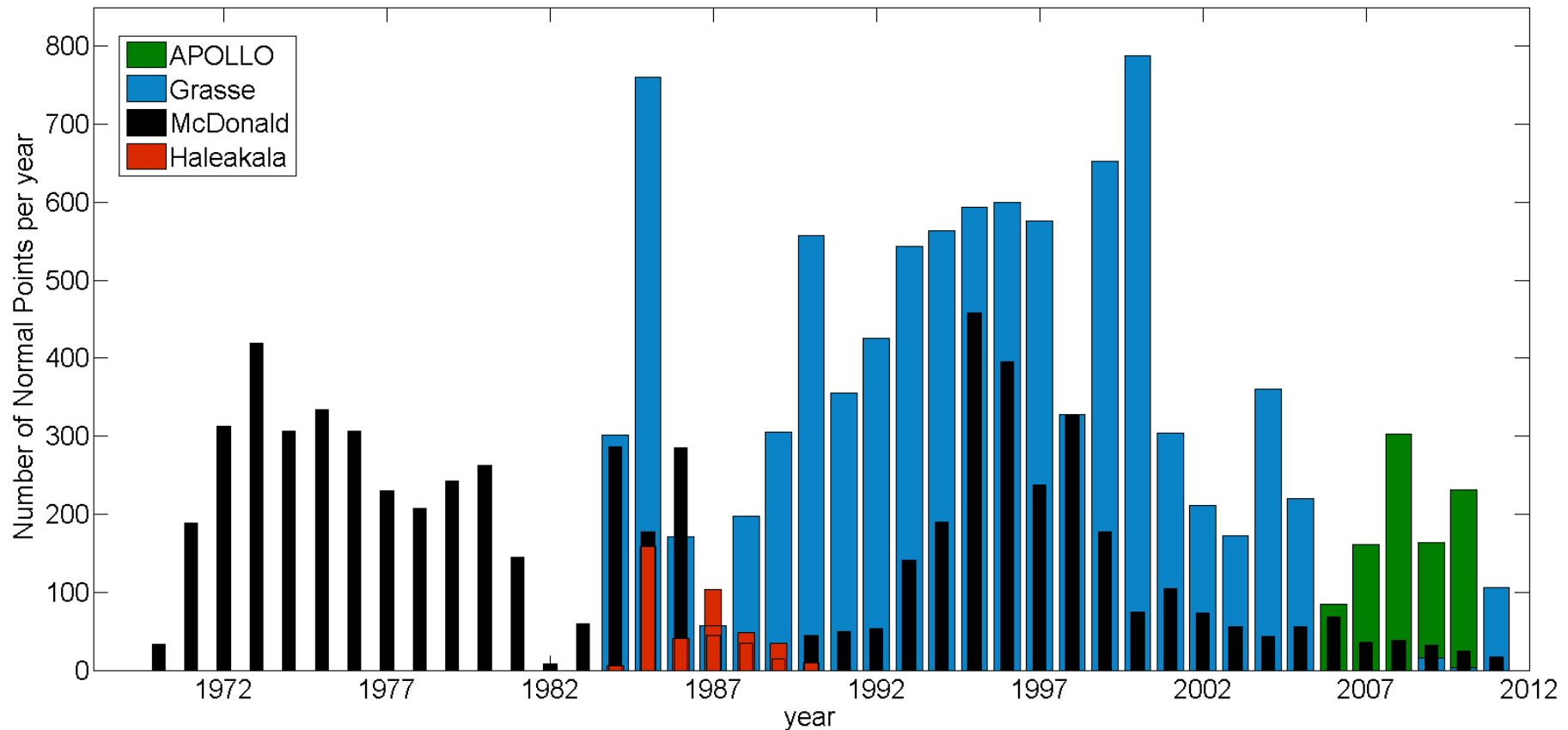


... and a few lunar tracks from

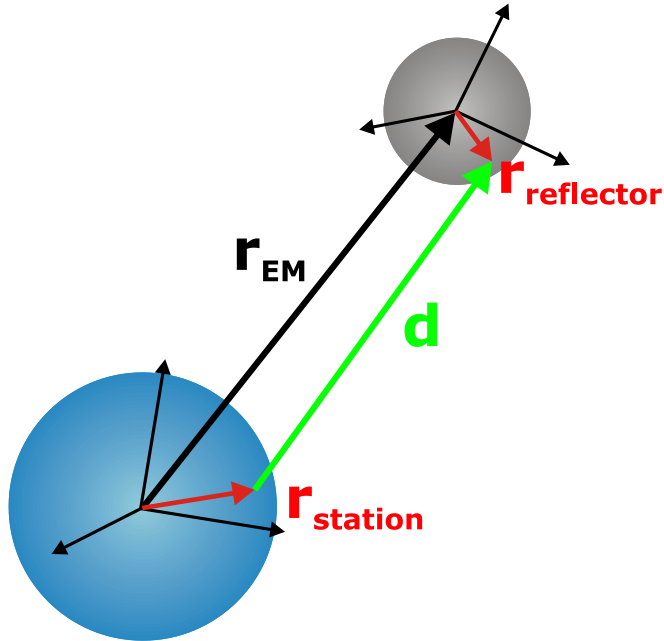
- Orroral
- Wettzell

Number of normal points

- 1970 - 2011: ca.17,000 normal points



Basic formulas



$$\mathbf{r}_{\text{reflector}} = \mathbf{R}^{\text{moon}} \mathbf{r}_{\text{reflector}}^{\text{SRF}}$$

$$\mathbf{r}_{\text{station}} = \mathbf{R}^{\text{earth}} \mathbf{r}_{\text{station}}^{\text{ITRF}} =$$

$$\mathbf{r}_{\text{station}} = \mathbf{S}(x_p, y_p, \text{UT1}) \mathbf{N} \mathbf{P} \mathbf{B} \mathbf{r}_{\text{station}}^{\text{ITRF}}$$

- Basic equation

$$d = \left| \mathbf{r}_{EM} - \mathbf{r}_{\text{station}} + \mathbf{r}_{\text{reflector}} \right| + c \Delta \tau \approx c \frac{\tau}{2}$$

- Barycentric ecliptical system J2000
- Weighted least-squares adjustment

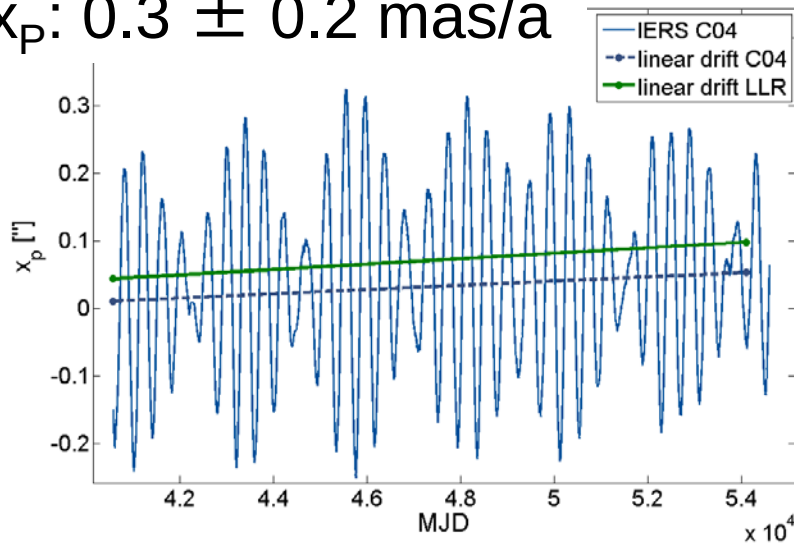
$$\tau + v = f(p_0) + \left. \frac{\partial f}{\partial p} \right|_{p_0} \Delta p$$

τ	Observations	$\frac{\partial f}{\partial p}$	Part. derivatives
v	Residuals		
p_0	Parameters – initial values		
Δp	Unknown increments		

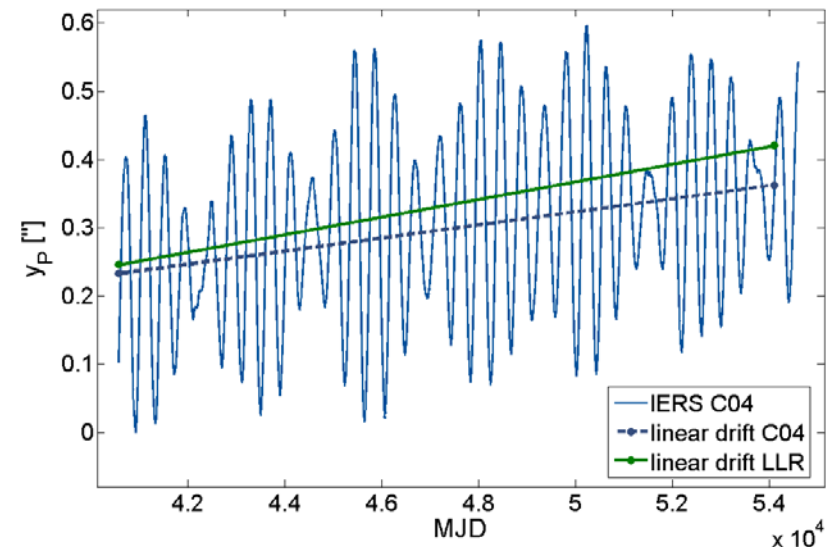
Linear drift in pole coordinates

- Differences between linear drift from IERS C04 and LLR for time span 1970-2008

$x_p: 0.3 \pm 0.2 \text{ mas/a}$



$y_p: 1.2 \pm 0.3 \text{ mas/a}$



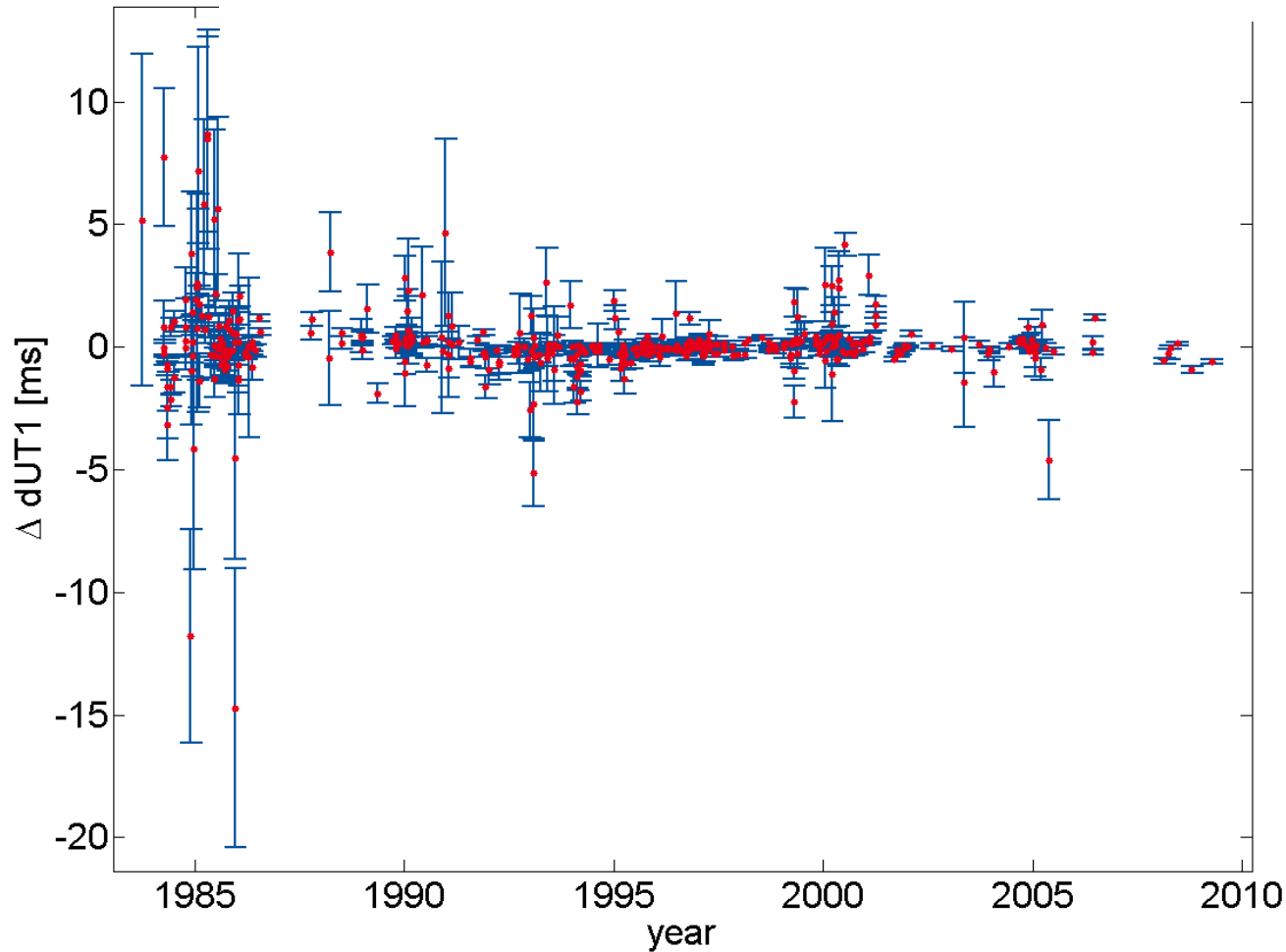
- Combined linear drift from LLR: $4.9 \pm 0.3 \text{ mas/a}$
- Linear drift from the combination of space geodetic techniques (Gross & Vondrák 1999): $4.123 \pm 0.002 \text{ mas/a}$ for time span 1976.7-1997.1

Earth rotation ΔUT

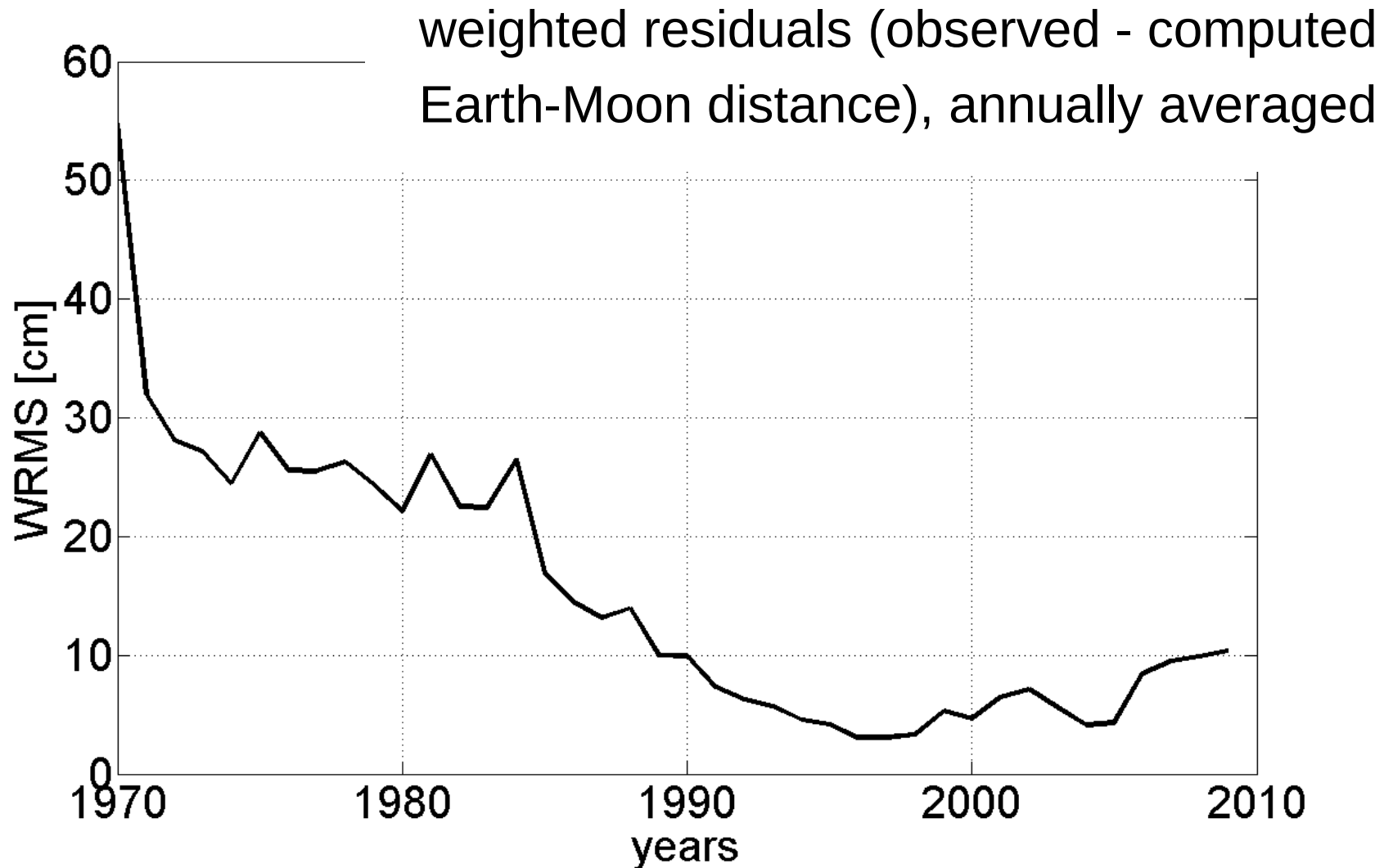
- Determination of UT1-UTC in global LLR adjustment
- Correction values to introduced UT1-UTC from the IERS C04 series were fitted
- Constraint that at least 10 LLR NP must be available for every considered night

Earth rotation ΔUT

Until 1987 the error bars in the order of ms only

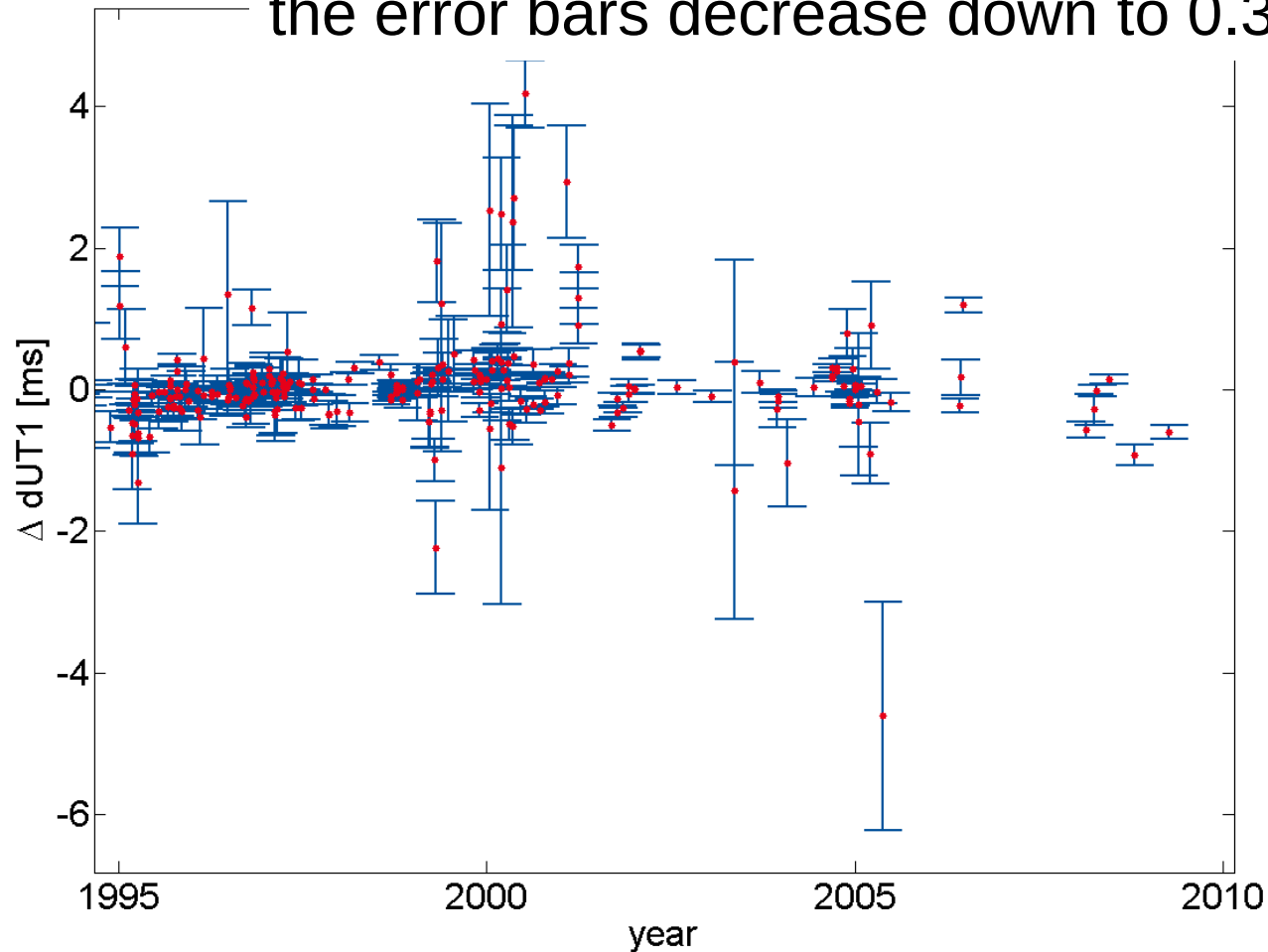


Weighted annual residuals



Earth rotation ΔUT

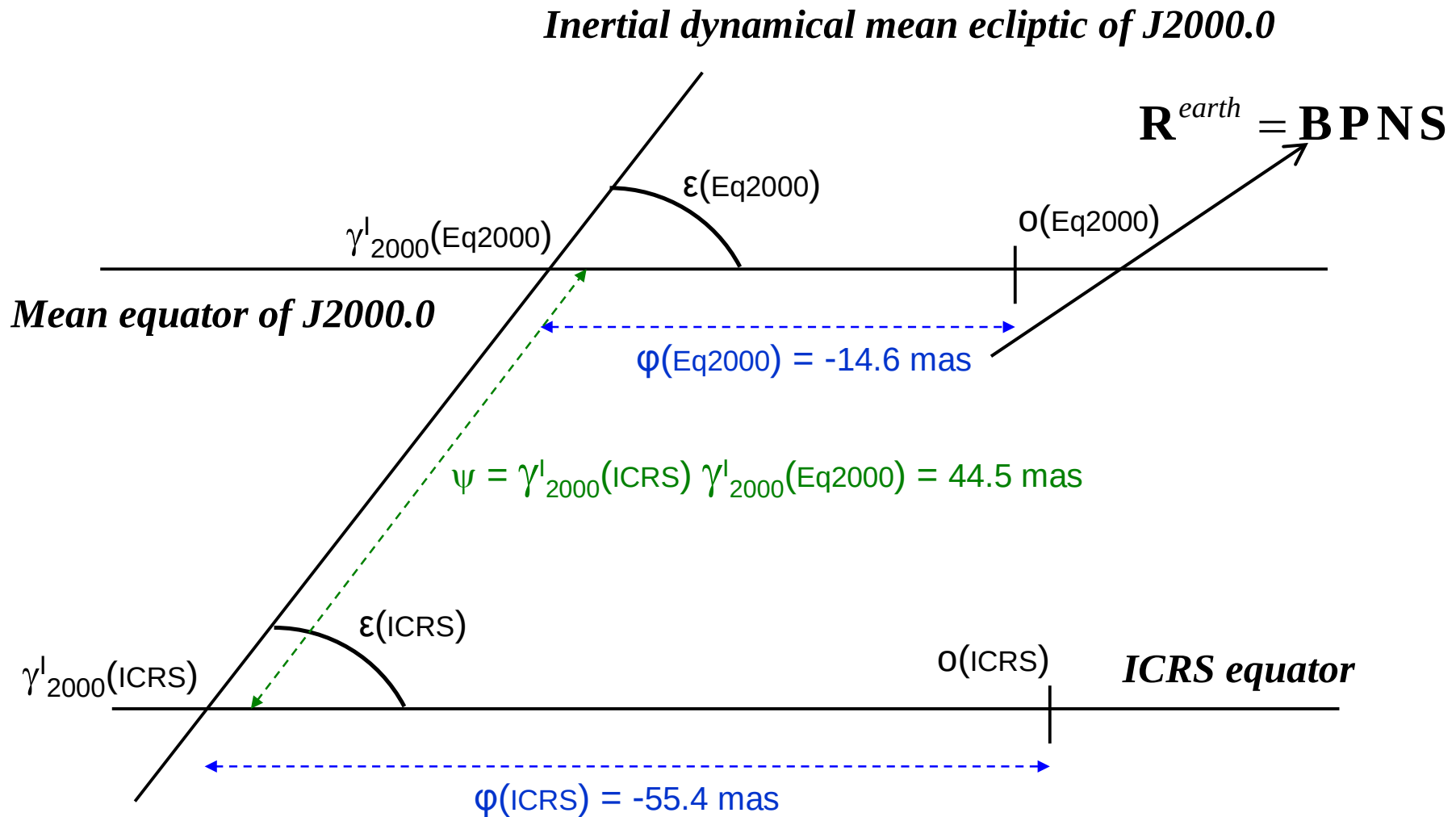
After 1988, when the data quality increased, the error bars decrease down to $0.3 \mu s$



Celestial reference frame

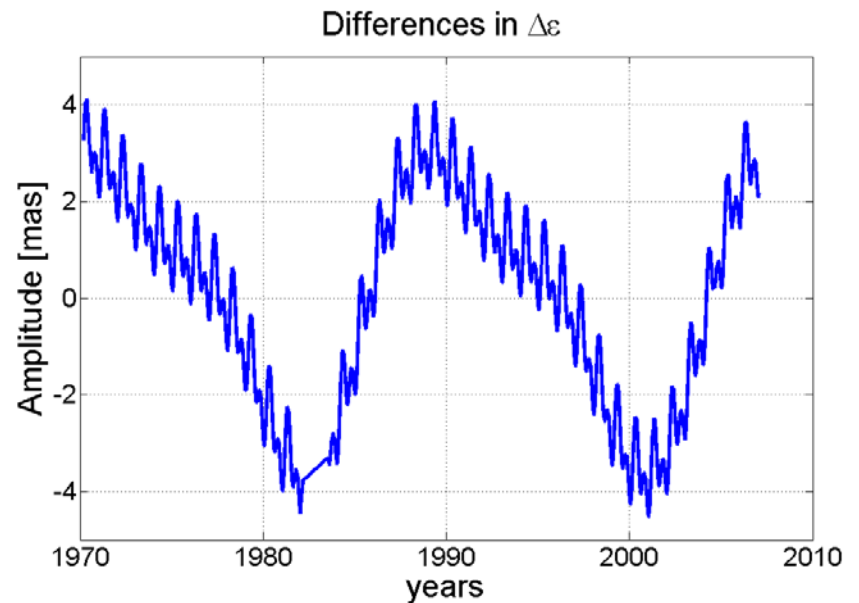
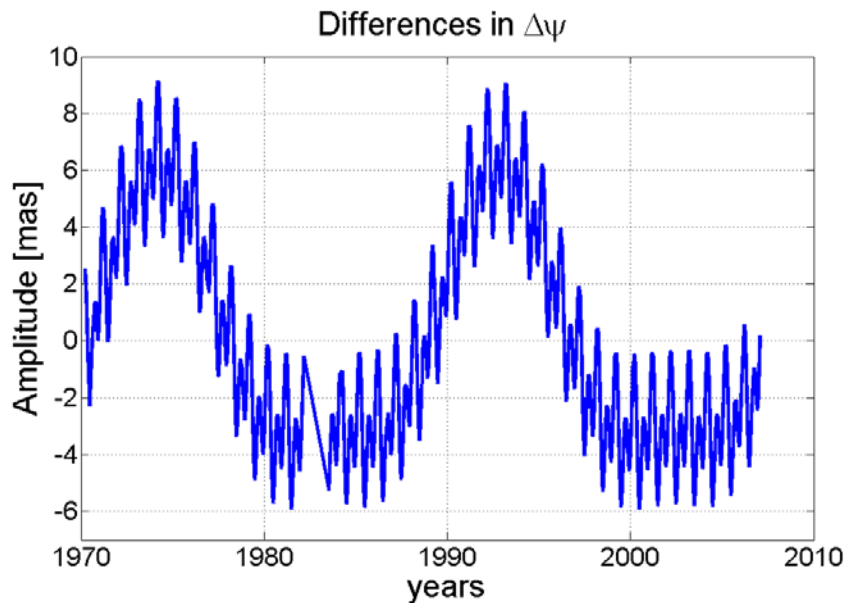
- Alignment of planetary/lunar and ICRS ephemerides < 1 mas
- VLBI and LLR give mean equinox J2000 (angles φ and ε)
- Mean inertial dynamic ecliptic from LLR based solution
- PN according to equator (ICRS or dynamic), corresponding angles φ and ε are determined
- Arc between $\gamma_{2000}^I(\text{ICRS})$ and $\gamma_{2000}^I(\text{Eq2000})$ on the ecliptic 44.5 ± 0.3 mas
- $\varepsilon^{(\text{ICRS})} = 23^\circ 26' 21''.4110 \pm 0.1$ mas; $\varphi^{(\text{ICRS})} = -55.4 \pm 0.1$ mas
- $\varepsilon^{(\text{Eq2000})} = 23^\circ 26' 21''.4056 \pm 0.1$ mas; $\varphi^{(\text{Eq2000})} = -14.6 \pm 0.1$ mas
- Values adopted by IAU, IERS (2010)

Celestial reference frame



Nutation

- Differences in nutation angles $\Delta\psi$ and $\Delta\epsilon$:
MHB2000 model minus LLR result



$$\mathbf{R}^{earth} = \mathbf{BPN S}$$

Analysis of rotation biases

- Transformation between terrestrial and celestial systems

$$\mathbf{r}_{station}^{ICRF} = \mathbf{R}^{earth} \mathbf{r}_{station}^{ITRF} \quad \mathbf{R}^{earth} = \mathbf{B} \mathbf{P} \mathbf{N} \mathbf{S}$$

- Extension for further rotation

$$\mathbf{R}_{earth}^{earth} \mathbf{B} \mathbf{P} \mathbf{N} \mathbf{S}$$

$$\mathbf{\Omega} = \mathbf{D}_1(\Theta_x) \mathbf{D}_2(\Theta_y) \mathbf{D}_3(\Theta_z) \approx \begin{bmatrix} 1 & \Theta_z & -\Theta_y \\ -\Theta_z & 1 & \Theta_x \\ \Theta_y & -\Theta_x & 1 \end{bmatrix}$$

$$\Theta_i = \Theta_{i0} + \dot{\Theta}_i \Delta t$$

Analysis of rotation biases

- Earth orbit fixed
- fit for Ω rotation (constant term) $\rightarrow$ then fixed

$$\Theta_x = -6,2 \pm 0,4 \text{ mas} \quad \rightarrow \Delta\varepsilon$$

$$\Theta_y = 1,1 \pm 0,2 \text{ mas} \quad \rightarrow \square \Delta\psi$$

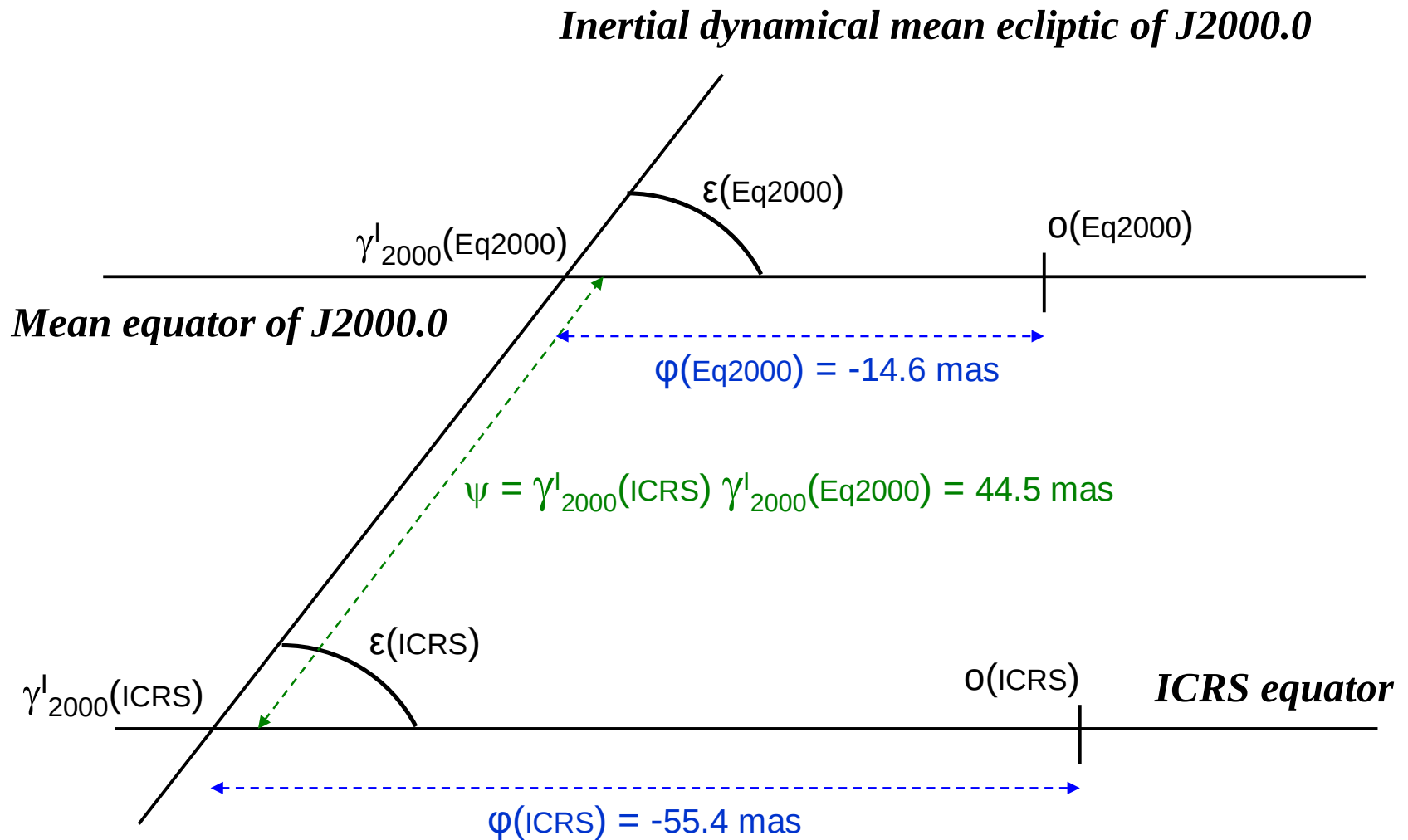
$$\Theta_z = 0,6 \pm 0,1 \text{ mas} \quad \rightarrow \Delta\varphi$$

- fit for $\dot{\Theta}_y$ and nutation, correlation up to 30 %

$$\dot{\Theta}_y = -0,6 \pm 0,1 \text{ mas/y}$$

$$\dot{P} = -1,6 \pm 0,2 \text{ mas/y}$$

Celestial reference frame



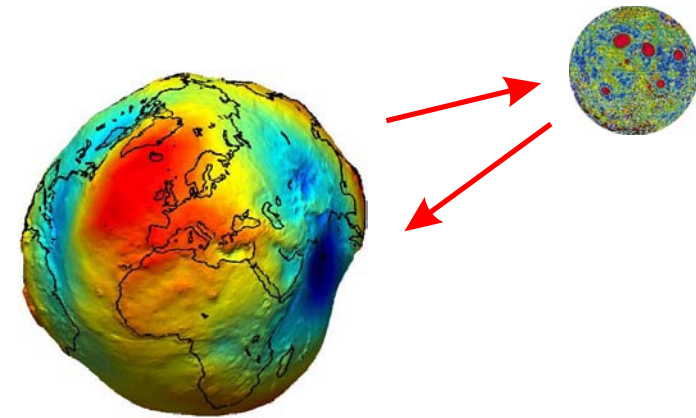
LLR tests of general relativity

Strong equivalence principle

$$\eta = (1 \pm 5) \times 10^{-4} \quad \left[\frac{M_G}{M_I} \right]_{SEP} - 1 = (-0.5 \pm 2.3) \times 10^{-13}$$

Temporal variation of the gravitational constant

$$G = G_0 \left(1 + \frac{\dot{G}}{G} \Delta t + \dots \right)$$
$$\frac{\dot{G}}{G} = (1 \pm 4) \times 10^{-13} \text{ yr}^{-1}$$



Factor 2 improvement due to refined modelling and more LLR data

Hofmann, Müller, Biskupek,
Astron. & Astroph., 2010

Conclusions

Open items (recommendations?)

- Added-value for long-term EOP monitoring by LLR?
- Celestial reference frame: better alignment of dynamic and ICRS equator possible/required?
- Change of time scale in planetary/lunar ephemeris ($T_{\text{eph}} - \text{TCB}$)?

Future steps

- Enhanced studies on ephemeris, earth rotation and relativity
- Software update to include and process novel observations from lunar transponders and orbiters (and VLBI in a more direct way)

Equivalence principle test with LLR

- Earth and Moon have a large amount of gravitational self energy
- use Earth-Moon system for testing strong equivalence principle (SEP) violation

$$\Delta r_{EM} = 12.8 \text{ m } \eta \cos D$$

- If $\eta \neq 0$
 - Different accelerations
 - Polarisation of lunar orbit

