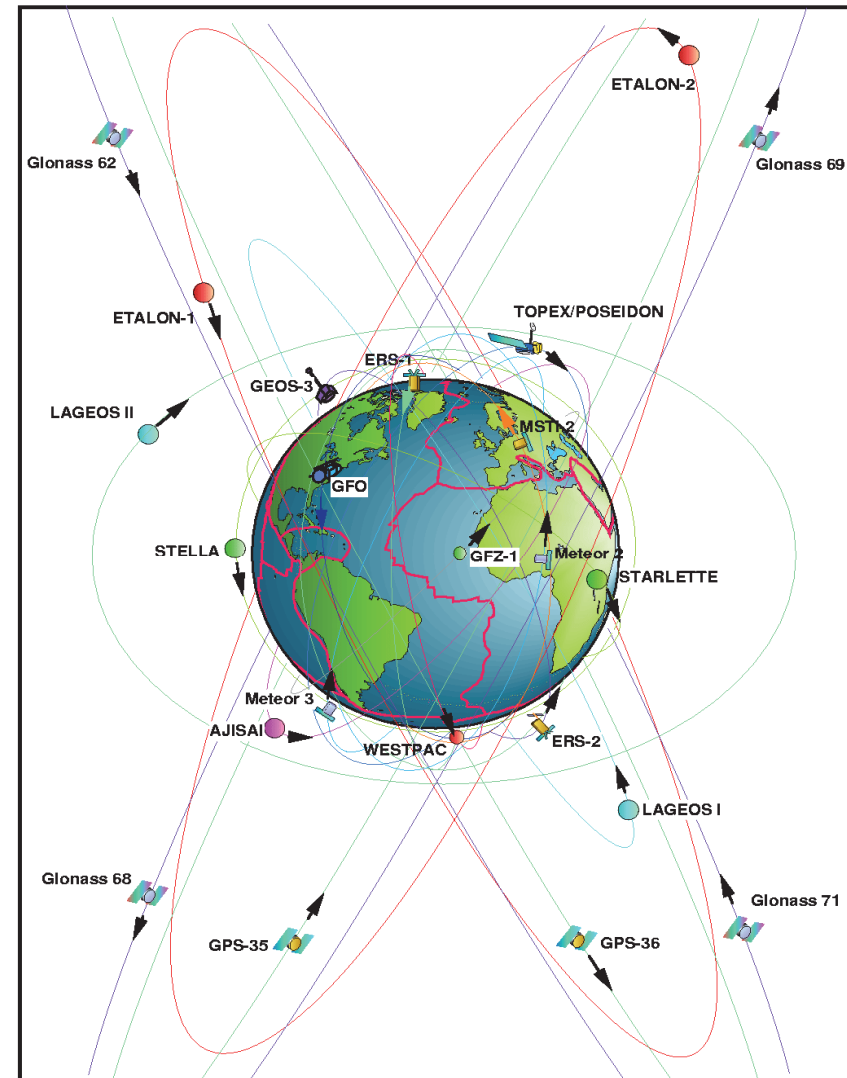




# The Scale of the Terrestrial Reference Frame from VLBI and SLR

J. C. Ries  
Center for Space Research  
The University of Texas at Austin

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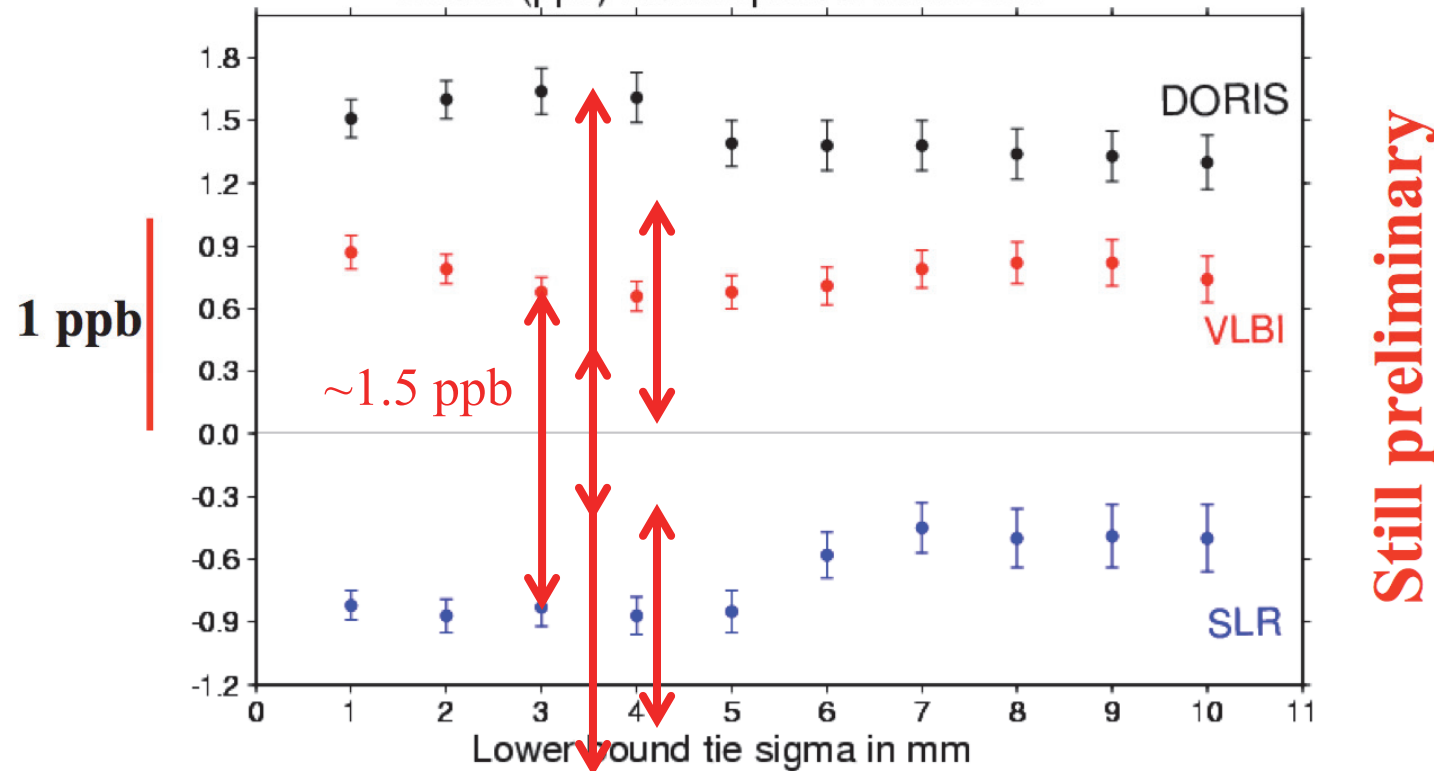
# Why is there a persistent scale difference between SLR and VLBI?



## Scale Factors of VLBI, SLR & DORIS wrt ITRF2008

Ten combination tests, varying the lower bound tie sigma

Scales (ppb) with respect to ITRF2008



3 mm as lower bound tie sigma is used in this analysis

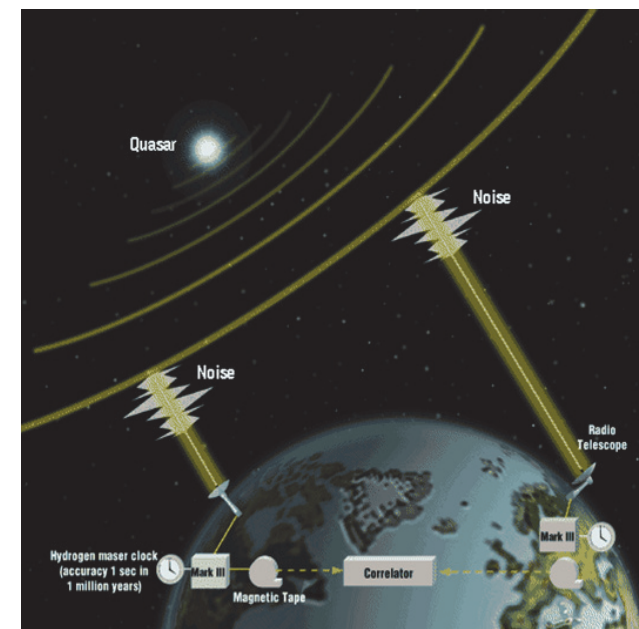
# Scale & Terrestrial Reference Frame (VLBI)



- The scale from VLBI is in theory determined only by the speed of light
  - The 'speed of light' depends on a correct modeling of relativity; is the 1991 consensus model correct? (relativistic contribution from Earth's mass to coordinate time transformation is 0.7 ppb)

$$t_{v_2} - t_{v_1} = \frac{\Delta T_{grav} - \frac{\hat{K} \cdot \vec{b}}{c} \left[ 1 - \frac{(1+\gamma)U}{c^2} - \frac{|\vec{V}_{\oplus}|^2}{2c^2} - \frac{\vec{V}_{\oplus} \cdot \vec{w}_2}{c^2} \right] - \frac{\vec{V}_{\oplus} \cdot \vec{b}}{c^2} (1 + \hat{K} \cdot \vec{V}_{\oplus} / 2c)}{1 + \frac{\hat{K} \cdot (\vec{V}_{\oplus} + \vec{w}_2)}{c}}$$

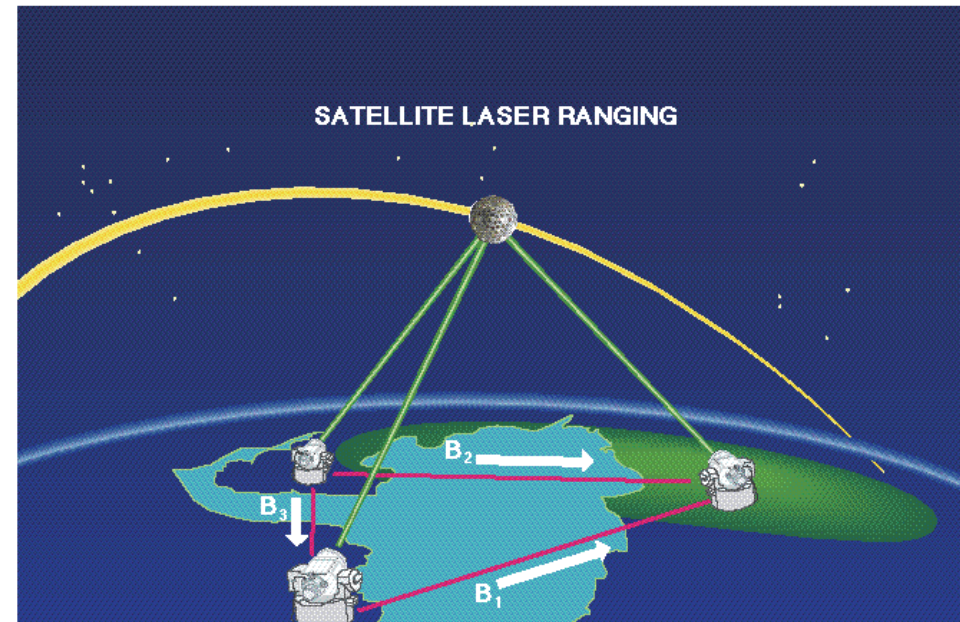
- In practice, troposphere estimation is required for radiometric techniques, which is highly correlated with station height and thus scale
  - What is the uncertainty in the zenith delay estimates? 1 mm, 3 mm...?
- Including gravitational antenna deflection might increase scale disparity further



# Scale & Terrestrial Reference Frame (SLR)



- The SLR scale is ultimately determined from the speed of light, orbital dynamics and the LAGEOS center of mass model (→GM)
- GM is usually held fixed; is the uncertainty in GM a limiting factor in the SLR scale?
- In 1992, GM estimated using 5 years of LAGEOS-1 data to determine value currently in use (based on CoM = 251 mm)
  - Must simultaneously estimate orbit, GM and all station heights to avoid locking in a priori scale
  - $GM = 398600.4415 \pm 0.0008 \text{ km}^3/\text{s}^2$  (TDT or TT value)
  - Assumed 2 cm biases plus a 'guesstimate' for troposphere error (0.2% or ~4-5 mm in zenith delay)

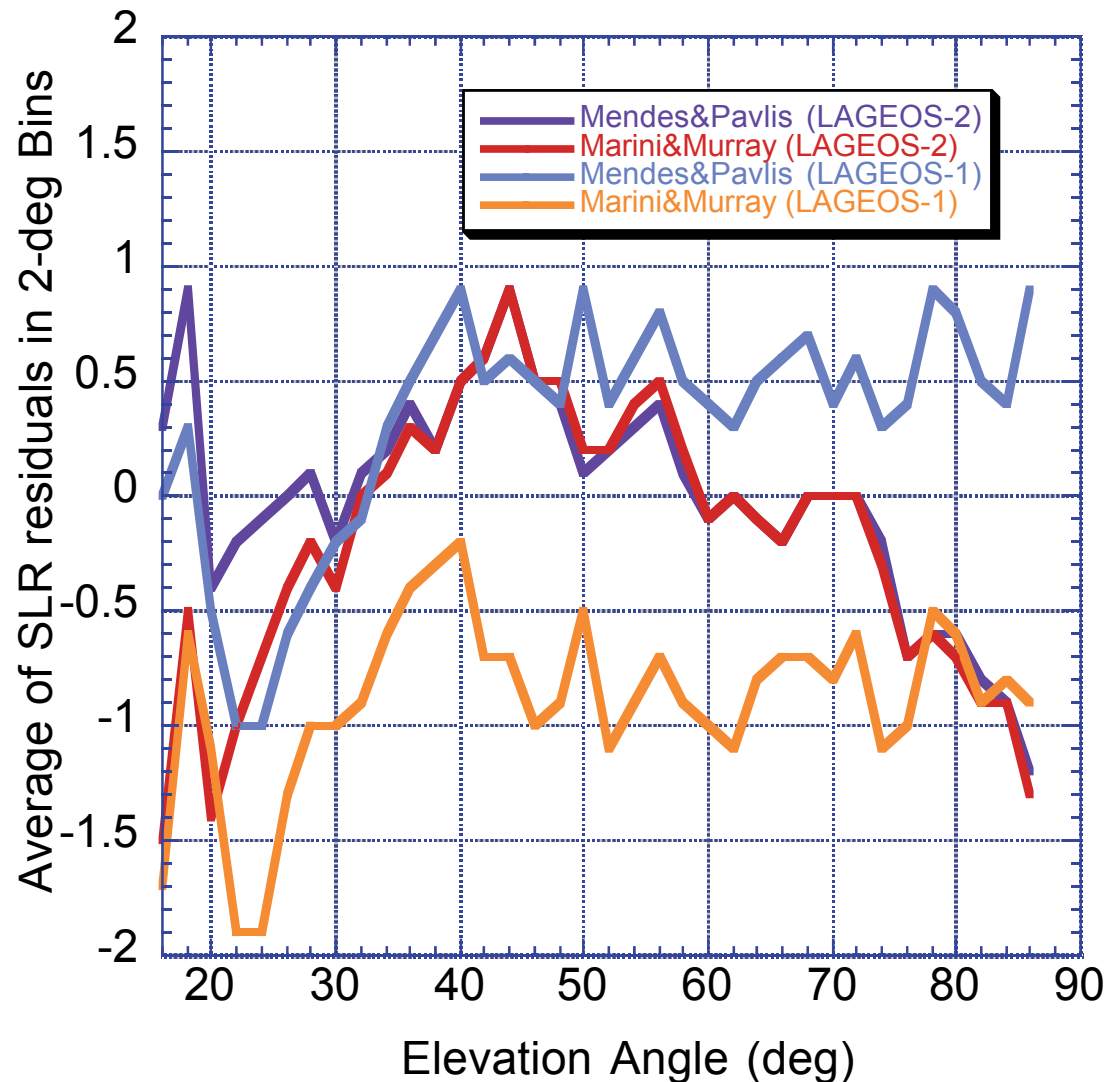


# Can we reduce uncertainty?



- In 2005, looked at updated tropospheric refraction model (Mendes et al., 2002; Mendes & Pavlis, 2004)
- Used 12 years of SLR data from LAGEOS-1 and LAGEOS-2 and considered 1 cm biases for each station (almost certainly pessimistic for better sites)
  - $GM = 398600.4415 \pm 0.00016 \text{ km}^3/\text{s}^2$  (using M&M refraction)
    - LAGEOS-1/2 differ by only 0.00013 (0.3 ppb), but most systematic errors are common
    - This uncertainty includes only random error (including bias)
- Repeated experiment with new refraction  $GM = 398600.4416 \pm 0.00016 \text{ km}^3/\text{s}^2$  (0.3 ppb change)
  - Change in terrestrial scale was only  $\sim 1.5 \text{ mm}$
  - Note that 1 ppb change in GM leads to 0.8 ppm change in TRF scale

# SLR Tropospheric Refraction Model Comparison for LAGEOS-1/2



All accepted SLR data for LAGEOS-1 and LAGEOS-2 since late 1992 (launch of L-2)

Residuals in 2 deg elevation bins, computing weighted average (only bins with > 1000 pts; most have 20000 to 30000 pts)

Significant variance reduction for Lageos-1 residuals

No change for Lageos-2

# GM estimation from SLR (total error estimate)

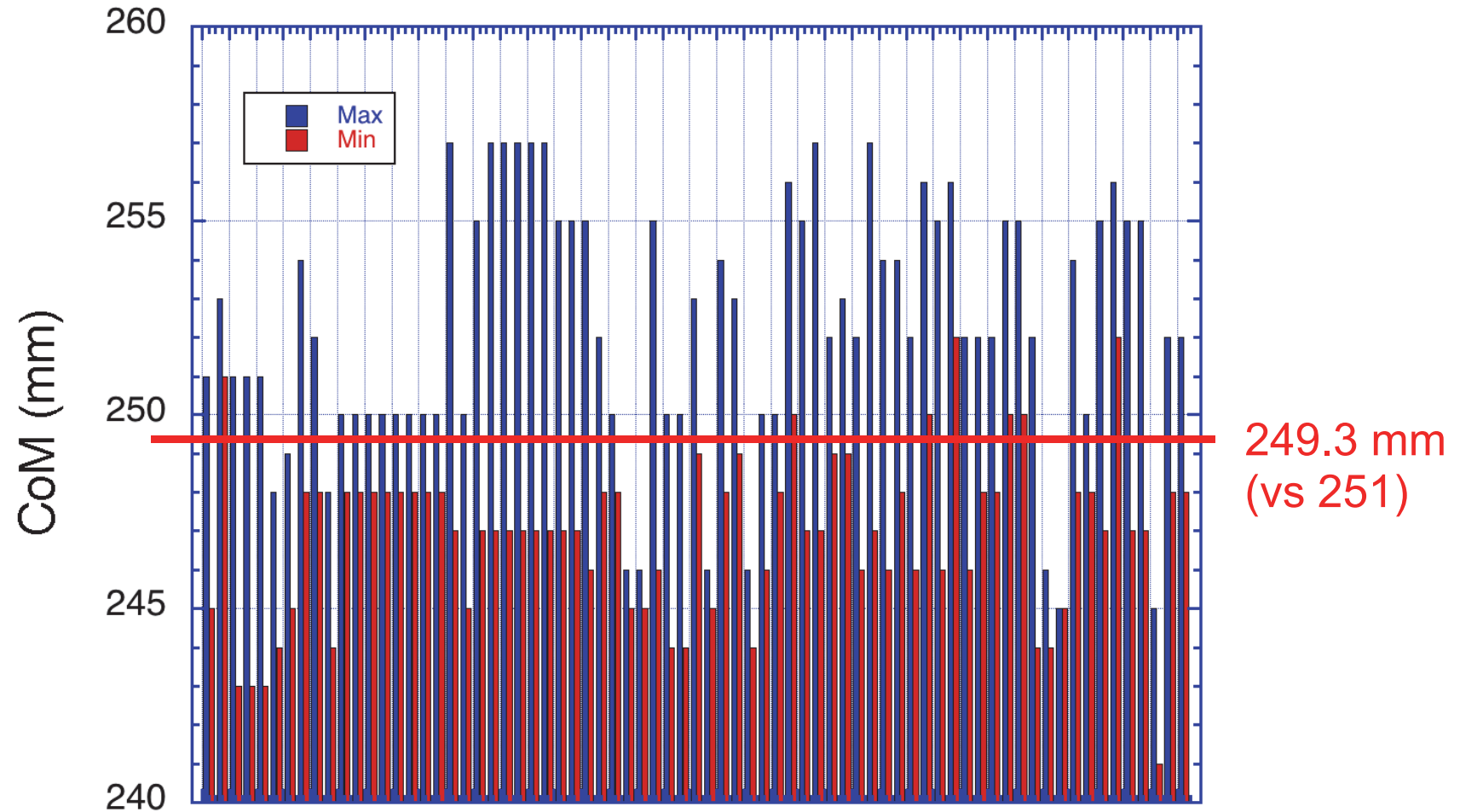
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- Assuming that errors in M&P are below the 1-mm level (zenith delay), contribution to GM error estimate appears to be small
- Consider contribution to error estimate from CoM model errors
  - It is easy to demonstrate that each 3 mm of CoM error leads to 1 ppb error in GM (for LAGEOS...differs by satellite altitude)
  - At the time,  $\sim 4$  mm in the CoM uncertainty seemed reasonable, if not a bit pessimistic
- Total uncertainty estimate for GM is thus dominated by CoM uncertainties
  - $GM = 398600.4416 \pm 0.0004 \text{ km}^3/\text{s}^2$  (adopting M&P model)
  - Change was too small to justify changing GM, though the uncertainty could be reduced from 2 ppb to 1 ppb
- With 20 years of LAGEOS data and in light of new modeling for LAGEOS CoM, how is the estimate of GM affected? Can the uncertainty be further reduced?



# Station Dependent CoM model



- GM = 398600.4417 (standard CoM) vs 398600.4416 (new CoM)
  - Uncertainty  $\sim 0.0002$  (0.5 ppb) assuming 2 mm uncertainty for CoM



# Conclusions (1)

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- Any determination of GM must include consideration of all systematic errors
  - All parameters (orbit, GM, and station heights) need to be free to adjust to avoid building in a priori scale
  - Troposphere refraction model error seems to be small
  - Center of mass offset model uncertainty dominates
- Using 20 years of L1/L2 data and allowing for the possibility of a persistent range bias of up to 5 mm in every station (probably still pessimistic for best stations), we get
  - Standard CoM:  $398600.4417 \pm 0.0003 \text{ km}^3/\text{s}^2$
  - Station-dependent CoM:  $398600.4416 \pm 0.0002 \text{ km}^3/\text{s}^2$ 
    - Assumes  $\sim 2$  mm uncertainty in new CoM model; assuming 1 mm uncertainty might be overly optimistic (at this level, effect of possible biases becomes comparable)

# Conclusions (2)

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- New estimate of GM would increase scale of SLR by only  $\sim 0.2$  ppb (and scale uncertainty due to GM is reduced to  $\sim 0.4$  ppb)
  - Other effects must be contributing to  $\sim 1.5$  ppm disparity between VLBI and SLR
- Scale offset between VLBI and SLR has persisted in spite of considerable improvements in data and analysis
  - Should re-examine VLBI consensus model (now 23 years old) to see if some small term has been overlooked
  - Concerned about effect of adding model for gravitational deflection (possibly increasing scale disparity)
  - SLR scale does not seem to be significantly biased due to GM error; can SLR analysis be improved in some other way?